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Damping of multiphoton Rabi oscillations and multiphoton Purcell effect

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Abstract

Damping of multiphoton Rabi oscillations due to a finite lifetime of photons is investigated using an algebraic method of the solution of the master equation. The time dependence of the probability of electron transitions between atomic levels is evaluated. In a regime of weak atom–photon coupling (a coupling constant smaller than the photon decay rate), the spontaneous emission of many photons from a single electron transition due to the interaction with a resonant light is found. Thus, a multiphoton counterpart of the Purcell effect is predicted.

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1. Introduction

The Purcell effect of the spontaneous emission from an atom being in resonance with a damped-photon field is utilized for building sources of highly coherent photons [1]. Much attention has recently been paid to this effect due to need of single-photon sources emitting almost indistinguishable photons in consecutive pulses for application in the quantum-information processing and in the quantum communication [2, 3]. One can expect that the damping of photons being in a multiphoton resonance with the atom would result in the spontaneous emission of many almost identical photons from a single-electron transition. Using such a multiphoton Purcell effect, one would be able to increase the intensity of the coherent radiation. This perspective is interesting for the emission from natural or artificial atoms interacting with a cavity mode, [4, 5], since the cavity capacity has to be small (the light has to be spatially confined) in order to ensure an effective coupling of the light to the matter.

Usually, the multiphoton transitions (especially multiphoton Rabi oscillations reported in [7] to be observed) are described assuming the Hamiltonian of the atom to be periodic in time (allowing the use of the Floquet theory) [8]. Thus, the electromagnetic field is treated as a classical one. However, in order to include a dissipation from the photon subsystem, the consideration of the interaction of the atom with a *quantum* field is necessary. Since perturbative methods are inapplicable to sequential photon processes, an algebraic solution of quantum dissipative equations is needed for their description. In the present paper, I apply an algebraic treatment of the master equation analysed in [9] equivalent to the formalism developed in [10] (known as 'thermo-field dynamics'). A description within a rigged-Fock-space formulation of the dissipative quantum mechanics, [11], is another proposal which has been found to reproduce some results of scattering matrix or Green-function methods [12, 13]. I include a finite lifetime of photons into the description of multiphoton transitions in the framework of the rigged-Fock-space formalism showing that the description is equivalent to the master-equation approach. The photon damping is found to lead to the damping of the multiphoton Rabi oscillations in a regime of strong electron-N-photon coupling (of a photon lifetime long compared to the inversion of an effective electron–photon interaction energy) or to the multiphoton Purcell effect in a relevant weak-coupling regime.

The present description can be used for different natural or artificial two-level systems. In particular, the quantum dots are recommended for observation of the highly coherent Purcell radiation because of their high stability compared to atoms, which results in a very small broadening of the emission line [3, 6]. However, since it is difficult to achieve an intense coherent light of a frequency smaller than the energy-level separation of the electron in artificial atoms, one can couple a quantum dot to an artificial molecule. This possibility was considered in [14].

In section 2, the model of an atom interacting with a photon field is presented and solved for the regime of the electron-N-photon resonance. In section 3, the consequences of the solution for the electron-transition probability are examined. Final conclusions are made in section 4.

2. Model and its solution

The multiphoton Rabi oscillations have been observed in a system of an atom interacting with an intense field periodic in time [7]. The dynamics of the system can be described using the second-quantization Hamiltonian,

$$\mathcal{H} = \sum_{j=1}^{2}[\omega_j + 2F_j\cos(\Omega t)]a_j^\dagger a_j + \left(Fa_1^\dagger a_2 + \text{H.c.}\right), \tag{1}$$

where $a_1^{(\dagger)}$, $a_2^{(\dagger)}$ denote annihilation (creation) operators of the electron on the first and second energy levels and Ω denotes the field frequency. The coupling coefficients F_j relate to the overlap of the electronic wavefunctions with the field. When we consider the coupling to a classical electro-magnetic field, the proportional to F_j terms in (1) result from the dependence of the electron kinetic energy on the vector potential. We do not specify the interaction constant F related to mutual overlapping of the wavefunctions of both electronic levels. For instance, this overlap can be introduced applying a static electric field when there is a non-zero transition dipole moment. Using the unitary transformation of the dynamical variables

$$a_j(t) = \exp(-\mathrm{i}2F_j\sin(\Omega t)/\Omega)\tilde{a}_j(t), \tag{2}$$

one finds that their equations of motion

$$\begin{aligned}\mathrm{i}\dot{\tilde{a}}_1 &= \omega_1\tilde{a}_1 + F\exp(\mathrm{i}2(F_1 - F_2)\sin(\Omega t)/\Omega)\tilde{a}_2\\ \mathrm{i}\dot{\tilde{a}}_2 &= \omega_2\tilde{a}_2 + F^*\exp(\mathrm{i}2(F_2 - F_1)\sin(\Omega t)/\Omega)\tilde{a}_1,\end{aligned} \tag{3}$$

belong to the class of equations solvable within a method developed in [8]. It was shown in [7] that (3) can describe the multiphoton Rabi oscillations when the coupling term represents a perturbation. Then, for $\omega_2 - \omega_1 = N\Omega$, one neglects in (3) terms representing non-conserving energy processes arriving at

$$\mathrm{i}\dot{\tilde{a}}_1 = \omega_1 \tilde{a}_1 + F_N\,\mathrm{e}^{\mathrm{i}N\Omega t}\tilde{a}_2, \qquad \mathrm{i}\dot{\tilde{a}}_2 = \omega_2 \tilde{a}_2 + F_N^*\,\mathrm{e}^{-\mathrm{i}N\Omega t}\tilde{a}_1, \tag{4}$$

where

$$F_N = F\left[\frac{1}{N!}\left(\frac{F_1 - F_2}{\Omega}\right)^N - \frac{1}{(N+1)!}\left(\frac{F_1 - F_2}{\Omega}\right)^{N+2} + \cdots\right]. \tag{5}$$

This approximation has left the degeneracy of the quasi-energy levels in the Floquet Hamiltonian of [7]. The solution of (4) is known to result in the oscillations of the probability of finding the electron in one of its two states with the frequency $\{(\omega_2 - \omega_1 - N\Omega)^2 + 4|F_N|^2\}^{1/2}/2$.

In order to include the interaction of the photon field with an environment, we perform its quantization writing the Hamiltonian in the form

$$\mathcal{H} = \sum_{j=1}^{2}[\omega_j + F_j'(b + b^\dagger)]a_j^\dagger a_j + \Omega' b^\dagger b + \left(F a_1^\dagger a_2 + \mathrm{H.c.}\right). \tag{6}$$

Here $b^{(\dagger)}$ denotes the photon annihilation (creation) operator. We introduce a finite photon lifetime into the description writing the master equation (see the appendix). We intend to study quantum-transition probabilities using a Gamow algebra of operators [12, 15] whose equations of motion are found in the appendix to be similar to the equations of motion of Fock-space vectors. In the relevant rigged-Fock-space Hamiltonian

$$\mathcal{H} = \sum_{j=1}^{2}\left[\omega_j + F_j'\left(b_{\mathrm{out}} + b_{\mathrm{in}}^\dagger\right)\right]a_{j\mathrm{in}}^\dagger a_{j\mathrm{out}} + \Omega b_{\mathrm{in}}^\dagger b_{\mathrm{out}} + F a_{1\mathrm{in}}^\dagger a_{2\mathrm{out}} + F^* a_{2\mathrm{in}}^\dagger a_{1\mathrm{out}}, \tag{7}$$

the photon frequency Ω is complex, ($\Omega = \Omega' - \mathrm{i}\Omega''$), unlike in (1)–(5). The photon annihilation and creation operators b_{out} and $b_{\mathrm{in}}^\dagger$ commute to unity while the electron operators $a_{j\mathrm{out}}, a_{j\mathrm{in}}^\dagger$ can be considered as commuting or anti-commuting to unity operators since we assume that there is only one electron in the system. They create the Gamow vectors of decaying states vanishing (left vectors) or diverging (right vectors) with the increase of time.

We perform a canonical (non-unitary) transformation $\tilde{O} \equiv \Lambda O \Lambda^*$ of any Gamow-algebra operator O analogous to the transformation (2) in order to diagonalize the adiabatic part of the Hamiltonian. Here the operation $(\cdot)^* \equiv [(\cdot)^\dagger]'$ is defined as the conjunction of the Hermitian coupling—denoted by $(\cdot)^\dagger$ and of the complex conjugate of the particle frequencies—denoted by $(\cdot)'$. It takes the form $\Lambda = \mathrm{e}^s$, where

$$s = \sum_{j=1}^{2}\frac{F_j'}{\Omega}a_{j\mathrm{in}}^\dagger a_{j\mathrm{out}}\left(b_{\mathrm{out}} - b_{\mathrm{in}}^\dagger\right), \tag{8}$$

and thus $\Lambda^* = \mathrm{e}^{-s}$. Via the relations

$$\begin{aligned} &a_{j\mathrm{out}} = \mathrm{e}^{-\tilde{\eta}_j}\tilde{a}_{j\mathrm{out}}, \qquad a_{j\mathrm{in}}^\dagger = \tilde{a}_{j\mathrm{in}}^\dagger\,\mathrm{e}^{\tilde{\eta}_j}, \qquad \tilde{a}_{j\mathrm{out}}\tilde{a}_{j\mathrm{in}}^\dagger = a_{j\mathrm{out}}a_{j\mathrm{in}}^\dagger, \\ &b_{\mathrm{out(in)}}^{(\dagger)} = \tilde{b}_{\mathrm{out(in)}}^{(\dagger)} - \sum_{j=1}^{2}\frac{F_j'}{\Omega}\tilde{a}_{j\mathrm{in}}^\dagger\tilde{a}_{j\mathrm{out}}, \end{aligned} \tag{9}$$

where $\tilde{\eta}_j = -F_j'/\Omega\left(\tilde{b}_{\mathrm{out}} - \tilde{b}_{\mathrm{in}}^\dagger\right)$, and via the identities

$$a_{j\mathrm{out}}a_{j\mathrm{in}}^\dagger a_{j\mathrm{out}} = a_{j\mathrm{out}}, \qquad a_{j\mathrm{in}}^\dagger a_{j\mathrm{out}}a_{j\mathrm{in}}^\dagger = a_{j\mathrm{in}}^\dagger, \qquad a_{1\mathrm{out}}a_{2\mathrm{in}}^\dagger = a_{2\mathrm{out}}a_{1\mathrm{in}}^\dagger = 0 \tag{10}$$

satisfied for a one-electron system, we transform the Hamiltonian into

$$\mathcal{H} = \sum_{j=1}^{2} \tilde{\omega}_j \tilde{a}_{j\text{in}}^\dagger \tilde{a}_{j\text{out}} + \Omega \tilde{b}_{\text{in}}^\dagger \tilde{b}_{\text{out}} + F \tilde{a}_{1\text{in}}^\dagger \, \mathrm{e}^{\tilde{\eta}_1 - \tilde{\eta}_2} \tilde{a}_{2\text{out}} + F^* \tilde{a}_{2\text{in}}^\dagger \, \mathrm{e}^{\tilde{\eta}_2 - \tilde{\eta}_1} \tilde{a}_{1\text{out}}. \quad (11)$$

Here $\tilde{\omega}_j = \omega_j - F_j'^2/\Omega$. Similarly as in the above 'semi-classical' treatment (4)–(5), we reduce the last term of (11) neglecting the processes non-conserving the energy. Furthermore, since, for $\mathrm{Re}(\tilde{\omega}_2 - \tilde{\omega}_1 - N\Omega) = 0$, the most probable energy-conserving process results from the interaction term of the form $F_N^- \tilde{b}_{\text{in}}^{\dagger N} \tilde{a}_{1\text{in}}^\dagger \tilde{a}_{2\text{out}} + F_N^+ \tilde{a}_{2\text{in}}^\dagger \tilde{a}_{1\text{out}} \tilde{b}_{\text{out}}^N$, where

$$F_N^- \equiv \frac{F}{N!} \left(\frac{F_1' - F_2'}{\Omega} \right)^N, \qquad F_N^+ \equiv \frac{F^*}{N!} \left(\frac{F_1' - F_2'}{\Omega} \right)^N, \quad (12)$$

we neglect other processes. The bigger N, the better the validity of this approximation.

Neglecting the perturbation term in the last of the following equations of motion (equation (13c))

$$\mathrm{i}\dot{\tilde{a}}_{1\text{out}} = \tilde{\omega}_1 \tilde{a}_{1\text{out}} + F_N^- \tilde{b}_{\text{in}}^{\dagger N} \tilde{a}_{2\text{out}} \quad (13a)$$

$$\mathrm{i}\dot{\tilde{a}}_{2\text{out}} = \tilde{\omega}_2 \tilde{a}_{2\text{out}} + F_N^+ \tilde{a}_{1\text{out}} \tilde{b}_{\text{out}}^N \quad (13b)$$

$$\mathrm{i}\dot{\tilde{b}}_{\text{out}} = \Omega \tilde{b}_{\text{out}} + F_N^- N \tilde{b}_{\text{in}}^{\dagger N-1} \tilde{a}_{1\text{in}}^\dagger \tilde{a}_{2\text{out}} \approx \Omega \tilde{b}_{\text{out}} \quad (13c)$$

one obtains an approximation relevant for studying the electron-level occupation (but not for studying the photon population). Therefore, equation (13c) becomes independent of (13a) and (13b). In order to diagonalize (13a) and (13b), we multiply them by $\tilde{b}_{\text{out(in)}}^{(\dagger)N/2}$ and find

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\left(\tilde{b}_{\text{out}}^{N/2} \tilde{a}_{1\text{out}}\right) = (\tilde{\omega}_1 + N\Omega/2)\left(\tilde{b}_{\text{out}}^{N/2} \tilde{a}_{1\text{out}}\right) + F_N^- \left(\tilde{b}_{\text{out}}^{N/2} \tilde{b}_{\text{in}}^{\dagger N/2}\right)\left(\tilde{a}_{2\text{out}} \tilde{b}_{\text{in}}^{\dagger N/2}\right) \quad (14a)$$

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\left(\tilde{a}_{2\text{out}} \tilde{b}_{\text{in}}^{\dagger N/2}\right) = (\tilde{\omega}_2 - N\Omega/2)\left(\tilde{a}_{2\text{out}} \tilde{b}_{\text{in}}^{\dagger N/2}\right) + F_N^+ \left(\tilde{b}_{\text{out}}^{N/2} \tilde{a}_{1\text{out}}\right)\left(\tilde{b}_{\text{out}}^{N/2} \tilde{b}_{\text{in}}^{\dagger N/2}\right). \quad (14b)$$

Two new operators defined as

$$\tilde{B}_j = A_{j1}\left(\tilde{b}_{\text{out}}^{N/2} \tilde{a}_{1\text{out}}\right) + A_{j2}\left(\tilde{a}_{2\text{out}} \tilde{b}_{\text{in}}^{\dagger N/2}\right) \quad (15)$$

satisfy the equations of motion

$$\mathrm{i}\dot{\tilde{B}}_j = \Omega_j \tilde{B}_j \quad (16)$$

with the transformation coefficients fulfilling the conditions

$$\begin{cases} A_{11}(\Omega_1 - \tilde{\omega}_1 - N\Omega/2) = A_{12} F_N^+ \frac{(\hat{n}+N)!}{(\hat{n}+N/2)!} \\ A_{12}(\Omega_1 - \tilde{\omega}_2 + N\Omega/2) = A_{11} F_N^- \frac{(\hat{n}+N/2)!}{\hat{n}!} \end{cases}, \qquad \begin{cases} A_{21}(\Omega_2 - \tilde{\omega}_1 - N\Omega/2) = A_{22} F_N^+ \frac{(\hat{n}+N)!}{(\hat{n}+N/2)!} \\ A_{22}(\Omega_2 - \tilde{\omega}_2 + N\Omega/2) = A_{21} F_N^- \frac{(\hat{n}+N/2)!}{\hat{n}!} \end{cases}, \quad (17)$$

where $\hat{n} \equiv \tilde{b}_{\text{in}}^\dagger \tilde{b}_{\text{out}}$. Here we used the identity $\tilde{b}_{\text{out}}^{N/2} \tilde{b}_{\text{in}}^{\dagger N/2} = (\hat{n} + N/2)!/\hat{n}!$. Solving (17), for

$$\Omega_1 = \frac{1}{2}\left\{ \tilde{\omega}_1 + \tilde{\omega}_2 - \left[(\tilde{\omega}_2 - \tilde{\omega}_1 - N\Omega)^2 + 4F_N^+ F_N^- \frac{(\hat{n}+N)!}{\hat{n}!} \right]^{1/2} \right\},$$
$$\Omega_2 = \frac{1}{2}\left\{ \tilde{\omega}_1 + \tilde{\omega}_2 + \left[(\tilde{\omega}_2 - \tilde{\omega}_1 - N\Omega)^2 + 4F_N^+ F_N^- \frac{(\hat{n}+N)!}{\hat{n}!} \right]^{1/2} \right\}, \quad (18)$$

one arrives at

$$A_{11} = A, \qquad A_{12} = A\frac{F_N^-(\hat{n}+N/2)!}{(\Omega_1 - \tilde{\omega}_2 + N\Omega/2)\hat{n}!},$$
$$A_{21} = B\frac{F_N^+(\hat{n}+N)!}{(\Omega_2 - \tilde{\omega}_1 - N\Omega/2)(\hat{n}+N/2)!}, \qquad A_{22} = B. \tag{19}$$

3. Results

The probability of the electron transition from the lower energy level to the higher one ($1 \rightarrow 2$) is proportional to the module square of the scalar product

$$\langle 0|b_{\rm out}^{M+N}(t)a_{1\rm out}(t)a_{2\rm in}^{\dagger}b_{\rm in}^{\dagger M}|0\rangle = \frac{M!}{(M+N/2)!}$$
$$\times\langle 0|\tilde{b}_{\rm out}^{M+N/2}(t)\big(\tilde{b}_{\rm out}^{N/2}\tilde{a}_{1\rm out}\big)(t)\,{\rm e}^{-\tilde{\eta}_1(t)}\,{\rm e}^{\tilde{\eta}_2}\big(\tilde{a}_{2\rm in}^{\dagger}\tilde{b}_{\rm out}^{N/2}\big)\tilde{b}_{\rm in}^{\dagger M+N/2}|0\rangle, \tag{20}$$

where M relates to the intensity of the photon field. Inverting the matrix of coefficients A_{ij}, we are able to evaluate its time dependence. Via

$$(A^{-1})_{11} = \frac{(\Omega_2-\omega_1-N\Omega/2)(\Omega_1-\omega_2+N\Omega/2)}{A\big[(\Omega_2-\omega_1-N\Omega/2)(\Omega_1-\omega_2+N\Omega/2)-F_N^+F_N^-(\hat{n}+N)!/\hat{n}!\big]},$$
$$(A^{-1})_{12} = -\frac{(\Omega_2-\omega_1-N\Omega/2)F_N^-(\hat{n}+N/2)!/\hat{n}!}{B\big[(\Omega_2-\omega_1-N\Omega/2)(\Omega_1-\omega_2+N\Omega/2)-F_N^+F_N^-(\hat{n}+N)!/\hat{n}!\big]},$$
$$(A^{-1})_{21} = -\frac{(\Omega_1-\omega_2+N\Omega/2)F_N^+(\hat{n}+N)!/(\hat{n}+N/2)!}{A\big[(\Omega_2-\omega_1-N\Omega/2)(\Omega_1-\omega_2+N\Omega/2)-F_N^+F_N^-(\hat{n}+N)!/\hat{n}!\big]},$$
$$(A^{-1})_{22} = \frac{(\Omega_2-\omega_1-N\Omega/2)(\Omega_1-\omega_2+N\Omega/2)}{B\big[(\Omega_2-\omega_1-N\Omega/2)(\Omega_1-\omega_2+N\Omega/2)-F_N^+F_N^-(\hat{n}+N)!/\hat{n}!\big]}, \tag{21}$$

we find

$$\langle 0|b_{\rm out}^{M+N}(t)a_{1\rm out}(t)a_{2\rm in}^{\dagger}b_{\rm in}^{\dagger M}|0\rangle = -\frac{M!}{(M+N/2)!}\exp(-{\rm i}(\tilde{\omega}_1+\tilde{\omega}_2+N\Omega)t)$$
$$\times\langle 0|\tilde{b}_{\rm out}^{M}(t)\,{\rm e}^{-\tilde{\eta}_1(t)}\,{\rm e}^{\tilde{\eta}_2}\tilde{b}_{\rm in}^{\dagger M}|0\rangle 2{\rm i}\sin\left\{\left[(\tilde{\omega}_2-\tilde{\omega}_1-N\Omega)^2+4F_N^+F_N^-\frac{(M+3N/2)!}{(M+N/2)!}\right]^{1/2}t/2\right\}$$
$$\times\frac{2F_N^-\frac{(M+3N/2)!}{(M+N/2)!}\left\{\tilde{\omega}_2-\tilde{\omega}_1-N\Omega+\left[(\tilde{\omega}_2-\tilde{\omega}_1-N\Omega)^2+4F_N^+F_N^-\frac{(M+3N/2)!}{(M+N/2)!}\right]^{1/2}\right\}}{\left\{\tilde{\omega}_2-\tilde{\omega}_1-N\Omega+\left[(\tilde{\omega}_2-\tilde{\omega}_1-N\Omega)^2+4F_N^+F_N^-\frac{(M+3N/2)!}{(M+N/2)!}\right]^{1/2}\right\}^2+4F_N^+F_N^-\frac{(M+3N/2)!}{(M+N/2)!}}. \tag{22}$$

The average on the right-hand side of (22) can be evaluated using the Feynman's disentanglement method relevant to an exactly solvable 'independent-boson model' [16, 17]. It was applied to a similar Gamow algebra in [13]. The result is

$$\langle 0|\tilde{b}_{\rm out}^{M}(t)\,{\rm e}^{-\tilde{\eta}_1(t)}\,{\rm e}^{\tilde{\eta}_2}\tilde{b}_{\rm in}^{\dagger M}|0\rangle = \exp\left\{-{\rm i}M\Omega t-\frac{F_1'^2+F_2'^2}{2\Omega^2}+\frac{F_1'F_2'}{\Omega^2}\,{\rm e}^{-{\rm i}\Omega t}\right\}$$
$$\times\sum_{j=0}^{M}\frac{(F_1'\,{\rm e}^{{\rm i}\Omega t}-F_2')^j(-F_1'\,{\rm e}^{-{\rm i}\Omega t}+F_2')^j}{\Omega^{2j}}\frac{M!}{j!}\binom{M}{j}. \tag{23}$$

Let us consider first the strong-coupling regime $[{\rm Im}(\tilde{\omega}_2-\tilde{\omega}_1-N\Omega)]^2 \ll 4\left|F_N^+F_N^-\right|\frac{(M+3N/2)!}{(M+N/2)!}$ (the nomenclature is similar as, e.g. in [18]). In this case, the amplitude of the probability of finding the system with the electron created on the lower atomic level in the state with the electron on the higher level $P(t) = \left|\langle 0|b_{\rm out}^{M+N}(t)a_{1\rm out}(t)a_{2\rm in}^{\dagger}b_{\rm in}^{\dagger M}|0\rangle\right|^2 \big/ [M!(M+N)!]$ oscillates with the frequency $\left[\left|F_N^+F_N^-\right|\frac{(M+3N/2)!}{(M+N/2)!}\right]^{1/2}$. This frequency increases with the parameter of the photon-field intensity M. The damping rate of $P(t)$ equal to $-{\rm Im}[\tilde{\omega}_1 + \tilde{\omega}_2 + (M+N)\Omega]$ is a sum of the decay rate of $(M+N)$-photon state and of the proper damping rate of the oscillations of the electron-level occupation equal to $-{\rm Im}(\tilde{\omega}_1 + \tilde{\omega}_2)$. From the form of (23), we predict the dephasing of the oscillations taking place in the characteristic time equal to Ω''^{-1}.

In the opposite case of the weak-coupling regime $[{\rm Im}(\tilde{\omega}_2 - \tilde{\omega}_1 - N\Omega)]^2 > 4\left|F_N^+F_N^-\right|\frac{(M+3N/2)!}{(M+N/2)!}$, another possible solution of the equations of motion is preferred. Taking

$$\Omega_1 = \Omega_2 = \frac{1}{2}\left\{\tilde{\omega}_1 + \tilde{\omega}_2 + \left[(\tilde{\omega}_2 - \tilde{\omega}_1 - N\Omega)^2 + 4F_N^+F_N^-\frac{(\hat{n}+N)!}{\hat{n}!}\right]^{1/2}\right\} \tag{24}$$

ensures that $P(t)$ does not diverge with time (and leads to $P(t) = 0$). We study the probability of the transition of the system from the state with the electron on the higher level $P'(t) = 1 - |\langle 0|b_{\rm out}^{M}(t)a_{2\rm out}(t)a_{2\rm in}^{\dagger}b_{\rm in}^{\dagger M}|0\rangle|^2/(M!)^2$. The scalar product in this expression takes the form

$$\begin{aligned}\langle 0|b_{\rm out}^{M}(t)a_{2\rm out}(t)a_{2\rm in}^{\dagger}b_{\rm in}^{\dagger M}|0\rangle &= \frac{(M!)^2}{[(M+N/2)!]^2}\\ &\quad\times\langle 0|\tilde{b}_{\rm out}^{M+N/2}(t)\big(\tilde{a}_{2\rm out}\tilde{b}_{\rm in}^{\dagger N/2}\big)(t)\,{\rm e}^{-\tilde{\eta}_2(t)}\,{\rm e}^{\tilde{\eta}_2}\big(\tilde{b}_{\rm out}^{N/2}\tilde{a}_{2\rm in}^{\dagger}\big)\tilde{b}_{\rm in}^{\dagger M+N/2}|0\rangle\\ &= \langle 0|\tilde{b}_{\rm out}^{M}(t)\,{\rm e}^{-\tilde{\eta}_2(t)}\,{\rm e}^{\tilde{\eta}_2}\tilde{b}_{\rm in}^{\dagger M}|0\rangle\exp\Bigg(-{\rm i}\frac{1}{2}\Bigg\{\tilde{\omega}_1 + \tilde{\omega}_2 + N\Omega\\ &\quad+\left[(\tilde{\omega}_2 - \tilde{\omega}_1 - N\Omega)^2 + 4F_N^+F_N^-\frac{(M+3N/2)!}{(M+N/2)!}\right]^{1/2}\Bigg\}t\Bigg).\end{aligned} \tag{25}$$

Evaluating the first factor on the right-hand side of (25), one finds

$$\begin{aligned}\langle 0|\tilde{b}_{\rm out}^{M}(t)\,{\rm e}^{-\tilde{\eta}_2(t)}\,{\rm e}^{\tilde{\eta}_2}\tilde{b}_{\rm in}^{\dagger M}|0\rangle &= \exp\left\{-{\rm i}M\Omega t - \frac{F_2'^2}{\Omega^2}(1-{\rm e}^{-{\rm i}\Omega t})\right\}\\ &\quad\times\sum_{j=0}^{M}\frac{F_2'^{2j}({\rm e}^{{\rm i}\Omega t}-1)^j(-{\rm e}^{-{\rm i}\Omega t}+1)^j}{\Omega^{2j}}\frac{M!}{j!}\binom{M}{j}.\end{aligned} \tag{26}$$

Neglecting the damping rate of (26) resulting from the decay of M-photon state, we find from $P'(t)$ the probability of occupation of the higher electron level. Its damping rate

$$\Gamma = \frac{(M+3N/2)!\left|F_N^+F_N^-\right|}{(M+N/2)!N\Omega''} \tag{27}$$

relates to the time of the emission of N photons from the atom $1/(2\Gamma)$. Thus, we predict a multiphoton Purcell effect. According to the classical Purcell result [1, 18], the emission time is inversely proportional to the photon lifetime and to the square of the effective constant of the electron–photon coupling. The emission time decreases with increase of the light intensity (with increase of the parameter M). Let us mention that, for the coherent sources of photons based on the multiphoton Purcell effect, the time of the electron decoherence resulting from (26) which is equal to Ω''^{-1} is expected to be long compared to the emission time.

4. Conclusions

I have considered the interaction of the two-level atom with a damped-photon field finding the damped oscillations of the electron-transition probability or the spontaneous multiphoton emission, depending on the ratio of the strength of the effective atom–photon interaction and the photon lifetime.

Unlike for linearly coupled optical systems displaying usual Rabi oscillations or Purcell effect, a dephasing of their multiphoton counterparts is inherent in our system. From (23) and (26), the dephasing takes the time equal to the twice of the photon lifetime. Let us note that both multiphoton effects disappear when the interaction constants F_1' and F_2' are equal. On the other hand, following (23), one can minimize the dephasing of Rabi oscillations when the parameter F_1' is equal to zero. Similarly, for $F_2' = 0$, the decoherence of the Purcell radiation disappears following (26).

In [7], there was proposed an atom for which the ratio F_1'/F_2' has been changed using the static electric field, which gives possibility of switching off the Rabi oscillations. Applying artificial atoms would enable one to control the coupling strength via the material of which the quantum dot is built.

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Appendix. Relation of the formalism to master equation

The knowledge of the relation of scalar products of Gamow vectors to scalar products of Fock-space vectors would make clear the interpretation of averages over the decaying states. We find it for the two-level atom interacting with a monochromatic light based on an algebraic method of solution of the master equation described in [9].

Let us consider the equation of motion of the density operator of our system $\mathrm{d}\rho/\mathrm{d}t = L\rho = \mathrm{i}^{-1}[H, \rho] + \mathcal{L}\rho$, where $\mathcal{L}$ denotes a dissipator. Following [9], for any operator A we define left (right) super-operators $A.$ $(.A)$ such that $A.\rho = A\rho$ $(.A\rho = \rho A)$, and

$$A.B. = AB., \qquad .A.B = .BA, \qquad .BA. = A..B. \tag{A.1}$$

Using them, one can write the Liouville operator as

$$L = \mathrm{i}^{-1}(H. - .H) + \mathcal{L}, \tag{A.2}$$

where, for our system in which the dissipation results in damping of photons,

$$\mathcal{L} = \Omega''\big(b..b^\dagger - \tfrac{1}{2}b^\dagger b. - \tfrac{1}{2}.b^\dagger b\big). \tag{A.3}$$

A canonical transformation of the creation (annihilation) super-operators

$$\begin{aligned} &\mathcal{A}_j^+ = a_j^\dagger. - .a_j^\dagger, && \mathcal{A}_j^- = a_j., \\ &\tilde{\mathcal{A}}_j^+ = .a_j - a_j., && \tilde{\mathcal{A}}_j^- = .a_j^\dagger, \\ &\mathcal{B}^+ = b^\dagger. - .b^\dagger, && \mathcal{B}^- = b., \\ &\tilde{\mathcal{B}}^+ = .b - b., && \tilde{\mathcal{B}}^- = .b^\dagger \end{aligned} \tag{A.4}$$

leads to the new form of the Liouvillian

$$L = \mathrm{i}^{-1}(\mathcal{H} - \tilde{\mathcal{H}}) + \mathrm{i}^{-1}(\mathcal{H}' - \tilde{\mathcal{H}}'), \tag{A.5}$$

where

$$\mathcal{H} = \sum_{j=1}^{2} [\omega_j + F_j'(\mathcal{B}^- + \mathcal{B}^+)]\mathcal{A}_j^+\mathcal{A}_j^- + \Omega\mathcal{B}^+\mathcal{B}^- + \left(F\mathcal{A}_1^+\mathcal{A}_2^- + F^*\mathcal{A}_2^+\mathcal{A}_1^-\right),$$
$$\mathcal{H}' = \sum_{j=1}^{2} F_j'\left[(\mathcal{B}^- + \mathcal{B}^+)\tilde{\mathcal{A}}_j^-\mathcal{A}_j^- + \tilde{\mathcal{B}}^-\left(\mathcal{A}_j^+\mathcal{A}_j^- + \tilde{\mathcal{A}}_j^-\mathcal{A}_j^-\right)\right]. \quad \text{(A.6)}$$

The operator $\tilde{\mathcal{H}}(\tilde{\mathcal{H}}')$ differs from $\mathcal{H}(\mathcal{H}')$ by changing $\mathcal{A}_j^\pm \leftrightarrow \tilde{\mathcal{A}}_j^\pm$, $\mathcal{B}^\pm \leftrightarrow \tilde{\mathcal{B}}^\pm$ and $\Omega \leftrightarrow \Omega^*$, $F \leftrightarrow F^*$. In the space of the vectors

$$\mathcal{R}_{mno;m'n'o'} \equiv \mathcal{A}_1^{+m}\mathcal{A}_2^{+n}\mathcal{B}^{+o}\tilde{\mathcal{A}}_1^{+m'}\tilde{\mathcal{A}}_2^{+n'}\tilde{\mathcal{B}}^{+o'}|0\rangle\langle 0|, \quad \text{(A.7)}$$

we consider those of $m' = n' = o' = 0$ (and $m + n = 1$ or $m + n = 0$). Since

$$\tilde{\mathcal{H}}\mathcal{R}_{mno;000} = \mathcal{H}'\mathcal{R}_{mno;000} = \tilde{\mathcal{H}}'\mathcal{R}_{mno;000} = 0, \quad \text{(A.8)}$$

we find $L\mathcal{R}_{mno;000} = \mathrm{i}^{-1}\mathcal{H}\mathcal{R}_{mno;000}$ and, thus, any combination of the vectors $\mathcal{R}_{mno;000}$ satisfies the Schrodinger-like equation with a non-Hermitian Hamiltonian however. On the other hand, since $\mathcal{R}_{mno;000} = a_1^{\dagger m}a_2^{\dagger n}b^{\dagger o}.|0\rangle\langle 0|$, the vectors $\mathcal{R}_{mno;000}|0\rangle$ create a basis of the Fock space. Thus, we have found the dynamics of the Fock-space vectors to be generated by a non-Hermitian Hamiltonian of the form similar to the Gamow Hamiltonian (7). All the scalar products of the Fock-space vectors can be expressed via scalar products of Gamow vectors. Similar to the above considerations show that $L\mathcal{R}_{000;mno} = -\mathrm{i}^{-1}\tilde{\mathcal{H}}\mathcal{R}_{000;mno}$ and the dynamics of the conjugated Fock vectors which can be decomposed into the basis of $\langle 0|\mathcal{R}_{000;mno}$ is generated by another non-Hermitian Hamiltonian.

Let us mention that introducing the operators (A.4) enables one to study the dissipative dynamics with one generator of the translation in time unlike some other methods using effective non-Hermitian Hamiltonians (the so-called quantum jump approach [19] including quantum trajectory [20] or stochastic Schrodinger-equation [21] methods). In that approach non-Hermitian Hamiltonians are generators of infinitesimal translation only and the evolution operator is an integral operator.

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